Simulating gravitational radiation from binary black hole mergers as LISA sources

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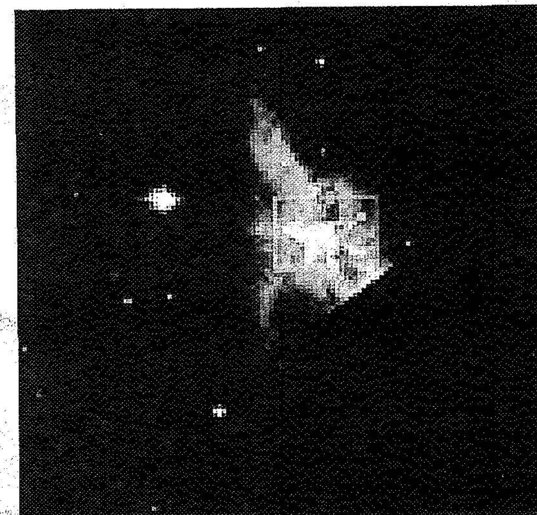
Massive Black Holes (MBHs)

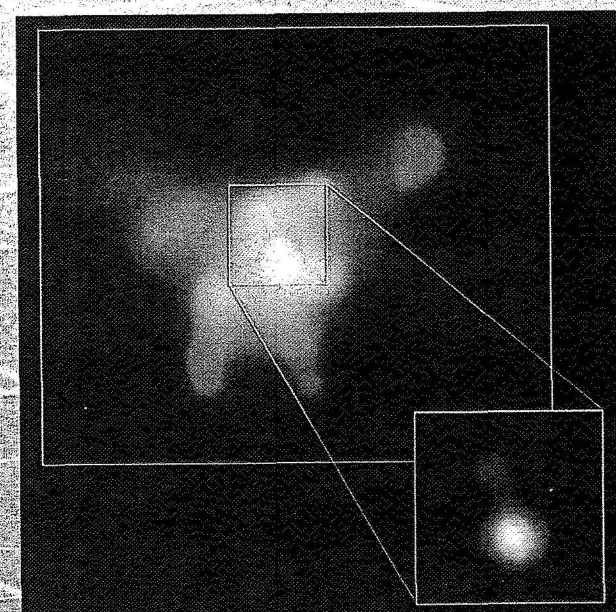
- Mass range: 10^3 $M_{Sun} < M <$ 10^9 M_{Sun}
- Theory
  - Cold Dark Matter (CDM) haloes condense from primordial anisotropies (eg $2 < z < 20$)
  - Centrally concentrated baryons lead to MBH
  - Many questions (e. g.)
    - Occupation fraction? Minimum halo mass?
    - Depend on cooling mechanisms / MBH formation process
  - Lower masses favored: 10^3 $M_{Sun} < M <$ 10^5 M_{Sun}
- Observation
  - MBHs seem to lie at the centers of virtually all galaxies with a bulge
  - Good evidence for MBHs with $M>$10^6 M_{Sun}
  - Some observations suggest a shortage of smaller MBH. (Could be a selection effect?)

MBH Binaries

- Binary Formation
  - Most galaxies are formed from the merger of smaller galaxies → merger of MBHs
  - Comparable mass systems likely
  - Binary observed in the galaxy NGC 6240
- Hardening
  - Dynamical friction may bring black holes close
  - Encounters with stars bridge 'last parsec'
  - Gravitational radiation energy loss dominates after binary is strongly hardened
  - Theory looks good for $M < 10^7 M_{Sun}$
- Expect for LISA
  - Event rate: 0.1 to 10000 per year
  - Best guess masses: 10^3-$10^5\ M_{Sun}$
  - Best evidence masses: 10^6-$10^7\ M_{Sun}$



(NGC 6240 -- Hubble)



(NGC 6240 -- Chandra)

Gravitational Waves from MBH Binaries

- Radiation Hardening
  - Hardening continues via radiative losses
- Inspiral
  - Slowly building amplitude
  - Many cycles per frequency octave
  - Analytic PN models apply
- Merger (aka 'Merger-Ringdown')
  - Approximately half of total energy release in last few cycles
  - Few cycles per octave
  - 3D numerical simulation of Einstein's field equations
    - Requires supercomputers
    - NEW Expect results!

Observing with LISA

- LISA Data analysis
  - Special features: LISA's motion, TDI, source confusion
  - Detection
    - Relatively easy
    - What astrophysics can we learn from event rate alone?
  - Source characterization
    - MBH-MBH → $(1+z)M_1$, $(1+z)M_2$, orientation, D_L, sky position
    - How can merger observations help?
- Science
  - Learn about structure formation process
    - Want to measure D_l
    - How much can we learn from event rate and red-shifted masses alone?
  - Testing GR
    - How to formulate a test based on comparable-mass mergers?
  - Electromagnetic MBH-MBH coincidence observations?
    - Merger forecasting?
    - Sky location ≤ degree^2
- Instrument
  - Design Considerations: Low-freq ($f < 10^{-4}$ Hz) sensitivity
  - Systematics, duty cycling, etc.

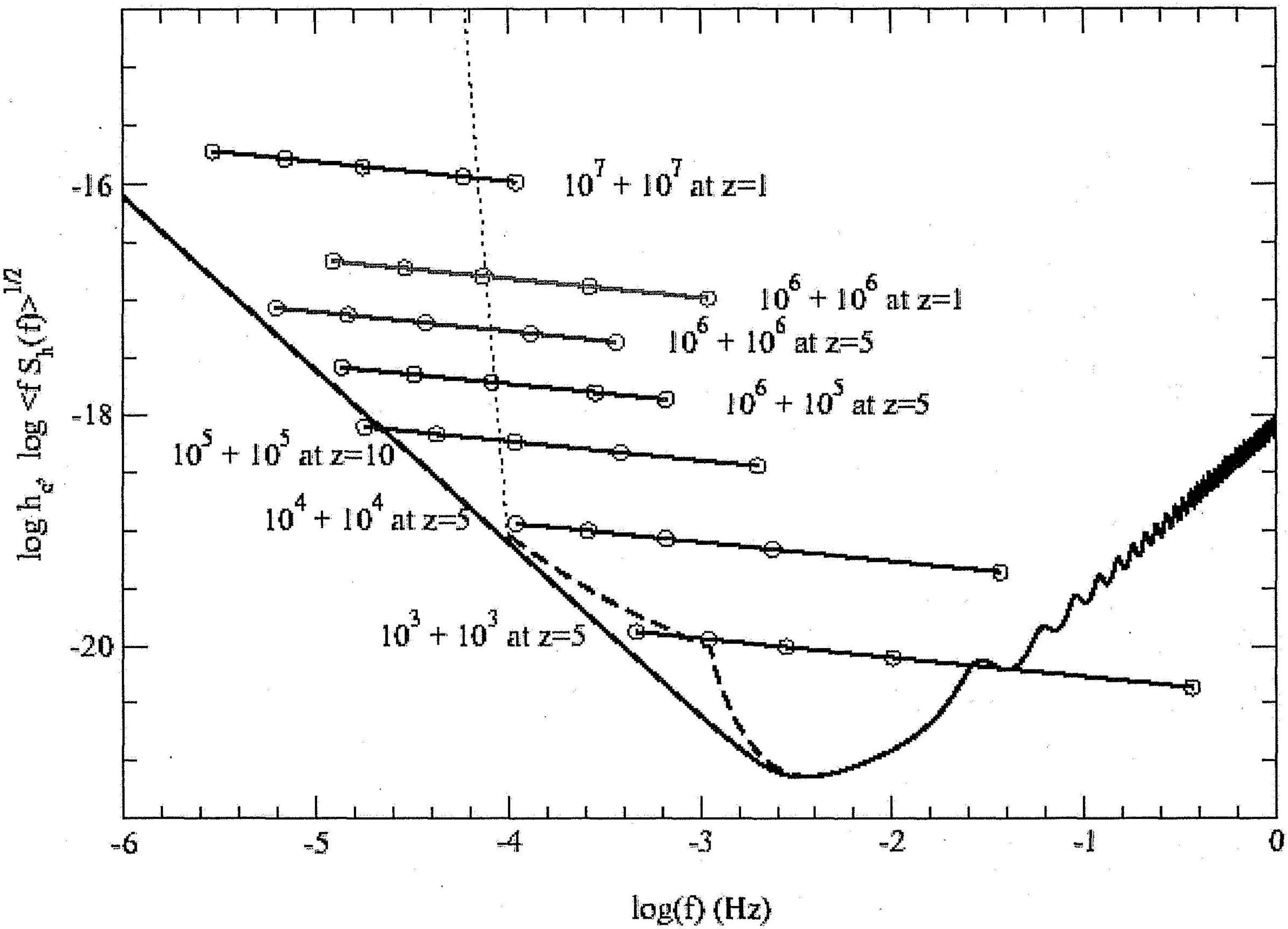
How LISA sees MBH binary mergers

- Instrumental strain noise spectrum $S_h(f)$
- Characteristic strain of GW signal $h_c = \frac{\sqrt{2}(1+z)}{\pi D_L(z)} \frac{dE}{df}[(1+z)f]$
- Expected signal-to-noise ratio (SNR) for obs of a chirping source using matched filtering

$$SNR^2 = \int_0^\infty \frac{df}{f} \frac{h_c^2(f)}{\langle f S_h(f) \rangle}$$

- For a detection, source must be within LISA's band of sensitivity at good SNR $h_c > \langle f S_h(f) \rangle$
- To measure D_L, and sky position
  - Annual motion modulations allow more accurate parameter estimates with inspiral observation
  - Generally need long (6 months) observations. (Careful studies underway.)
  - **Little work on merger observations.**

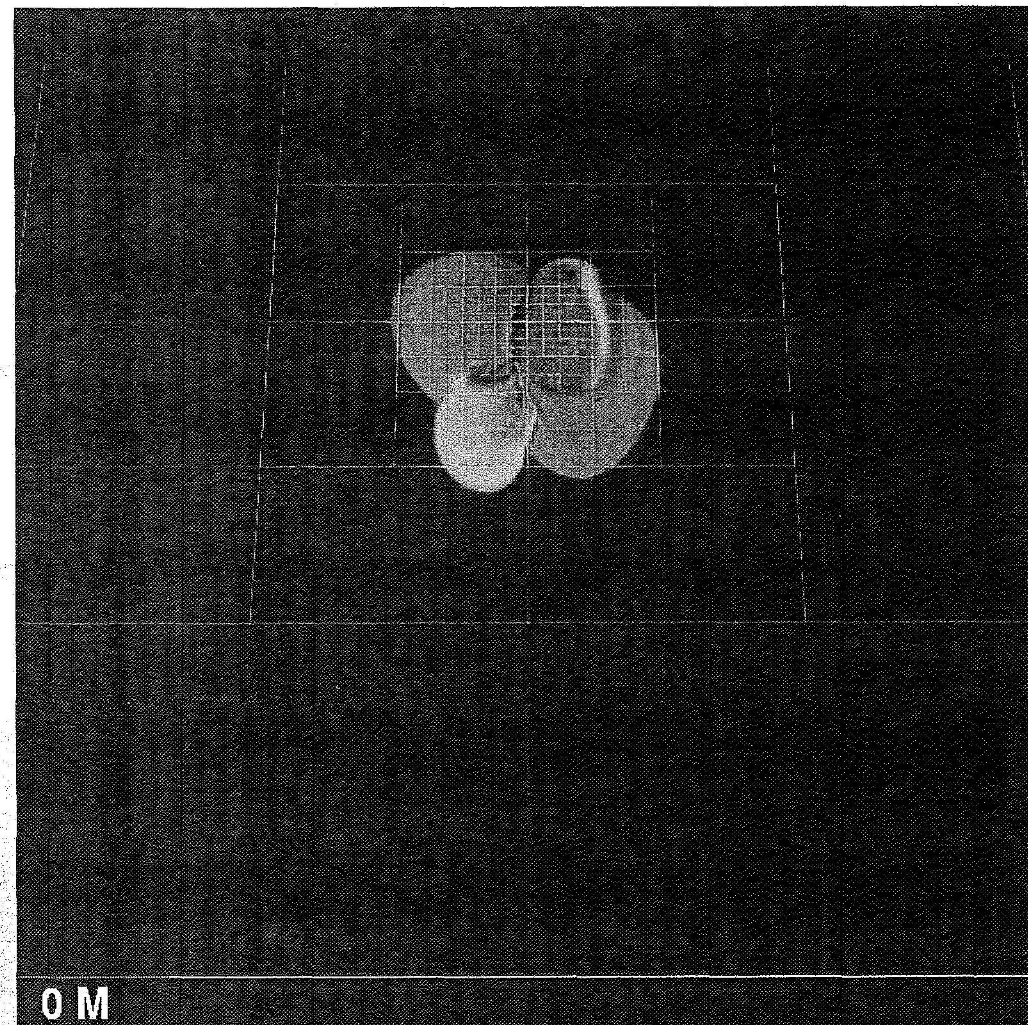
MBH binary inspirals to LISA



Symbols at 10 yrs, 1 yr, 1 mo, & 1 day before merger, and at the onset of merger

Numerical relativity simulations

- Einstein's equations on a supercomputer
  - 3+1 Formulation yields evolution problem
  - 3-D Non-linear system of ~20 variables.
  - Fields are also subject to 4 nonlinear constraints.
  - *E.g* 10000 CPU-hrs per simulation.
- No matter for Black Holes
  - Can assume *vacuum* GR
  - Every black hole system has mass M !
    - LISA: $M \sim 10^3\text{-}10^7\ M_{Sun}$
    - LIGO: $M \sim 20\text{-}80\ M_{Sun}$
  - *see also* Lousto's talk.



Numerical Relativity Challenges

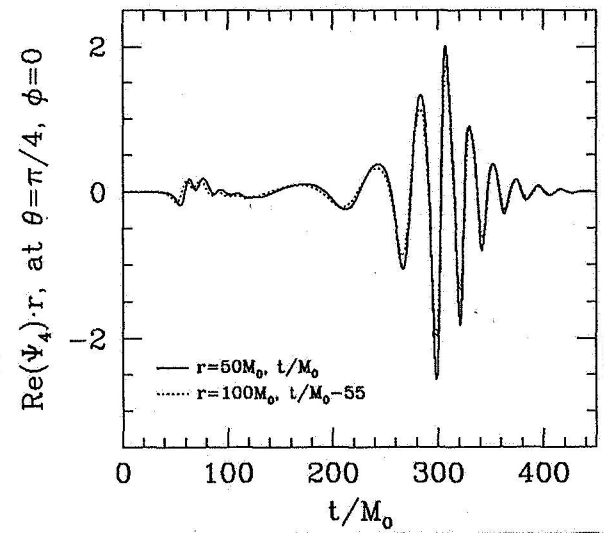
- **Formulation**
  - **A few years ago:**
    - ADM system was unstable
    - Black hole simulations blew-up by 30M
  - **Reformulated by adding new variables and constraint identities**
  - **Now several stable, strongly hyperbolic systems in use**
  - **Developing systems which control constraint evolution**
- **Treating black holes**
  - **Initial data**
    - Need initial values for *fields*
    - Must satisfy *constraints*
    - Need model to represent *astrophysical* system
  - **Singularities**
    - Now several tools available
    - Singularity avoiding time slices (lapse choice)
    - Excision / "puncture splitting", or ...

Numerical Relativity Challenges

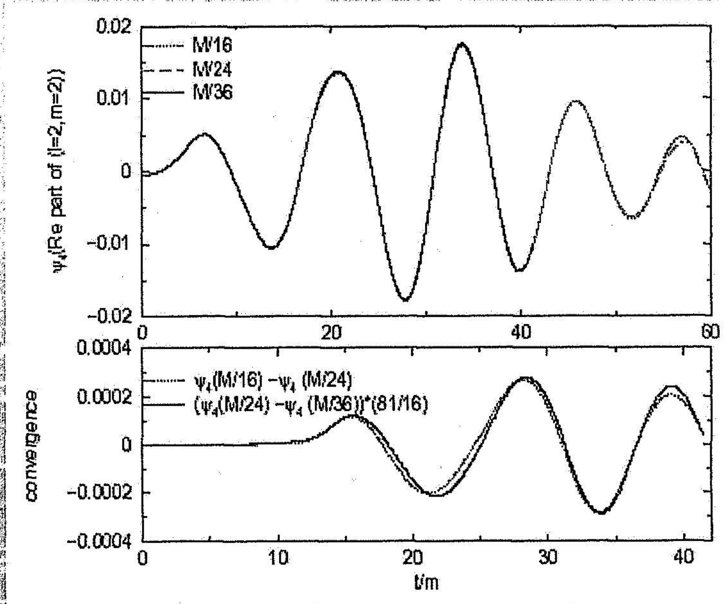
- Gauge Choice
  - Must dynamically specify a coordinate system
    - General covariance: anything goes!
    - But most choices are pathological
  - Recent years:
    - Have prescriptions good for single BH
    - Accuracy of binary BH simulations strongly sensitive
    - One question:
      - Let the black holes move?
      - Or fix to grid?
- Accuracy
  - A relatively new research area
  - Numerics
    - AMR (and FMR)
    - Higher-order finite differencing
    - Spectral methods
  - Boundary conditions
  - Constraints / Gauge choice / Initial Data ...

Recent Successes

- Three groups now calculating waveforms from orbiting black holes!
  - Waveforms show: $r\,\psi_4 = r\, d^2/dt^2(h_+ + i\, h_x)$
  - Typically l=2,m=2 (spin-weight= -2)
  - All allow moving black holes
  - Three unrelated codes
- Frans Pretorius
  (Pretorius, PRL, 95, 121101 (2005), gr-qc/0507014)
  - Generalized harmonic gauge/formulation
  - AMR
  - First waveforms from full-orbit simulations
  - Black hole excision
- UTB-Group (Campanelli, et al: gr-qc/0511048)
  - Specialized BSSN-Γ-Freezing gauge/form.
  - Fourth-order differencing methods
  - No excision
- Goddard-Group (Baker, et al, gr-qc/0511103)
  - Another specialized BSSN-Γ-Freezing gauge/form.
  - Adaptive Mesh Refinement (AMR)
  - No excision



(Frans Pretorius)



(UTB-Group)

Goddard Team

- GSFC Numerical Relativistic Astrophysics Group
  - Joan Centrella, John Baker (NASA/GSFC)
  - Dae-Il Choi (USRA), Jim van Meter, Michael Koppitz (National Research Council)
  - Breno Imbiriba, W. Darian Boggs, Stefan Mendez-Diez (University of Maryland)

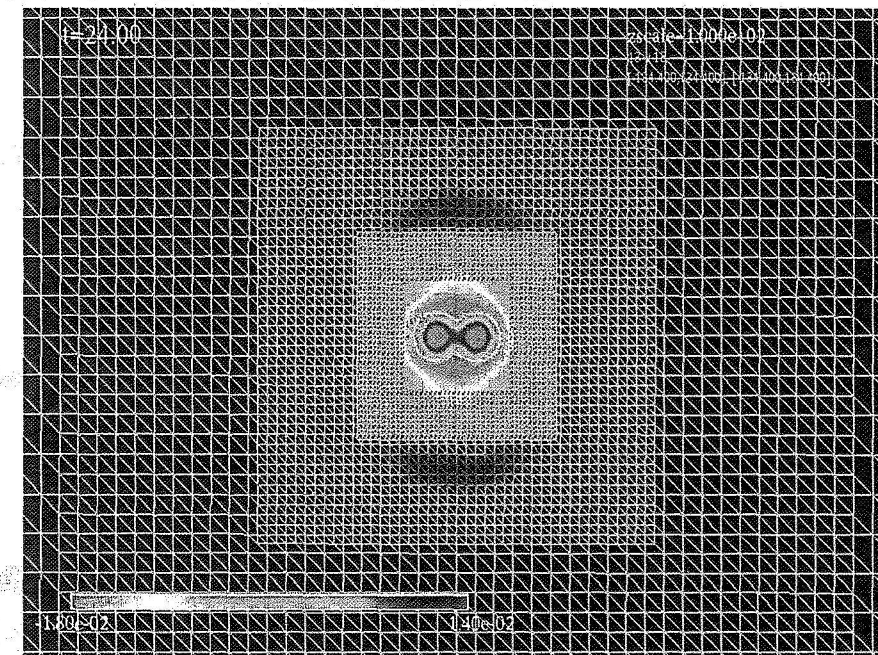
- Other collaborators
  - J. David Brown (North Carolina State Univ.)
  - David Fiske (DAC, formerly NASA/GSFC)
  - Kevin Olson (NASA/GSFC)

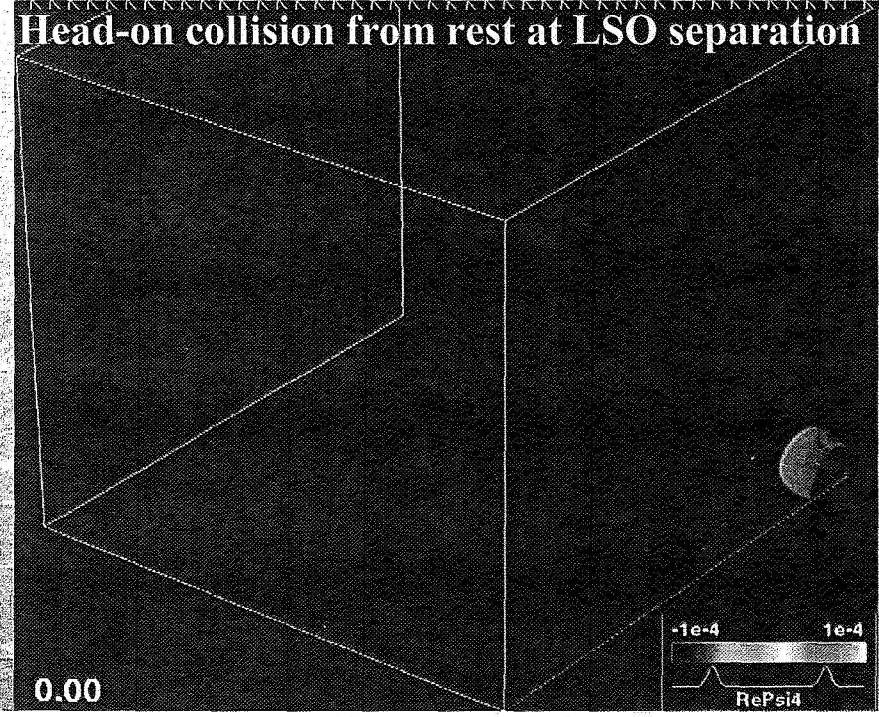
Binary black hole simulations at Goddard

- Basics
  - Use stable BSSN formulation
  - Singularity avoiding slicing
  - Black holes
    - Use “puncture” black hole treatment
    - With variations to allow black holes to move.
  - Use leading “Gamma-freezing” shift
    - Variations for moving black holes
- Numerics
  - Use nested mesh refinement boxes for:
    - large computational domain (scale 10^2 M)
    - With high resolution around black hole region (scale 1M)
    - Allows clean waveform studies
  - Extract ψ_4 waveforms
    - Spacetime curvature component
    - Corresponds to test-mass accelerations

$$\psi_4 = d^2/dt^2(h_+ + i\, h_x)$$

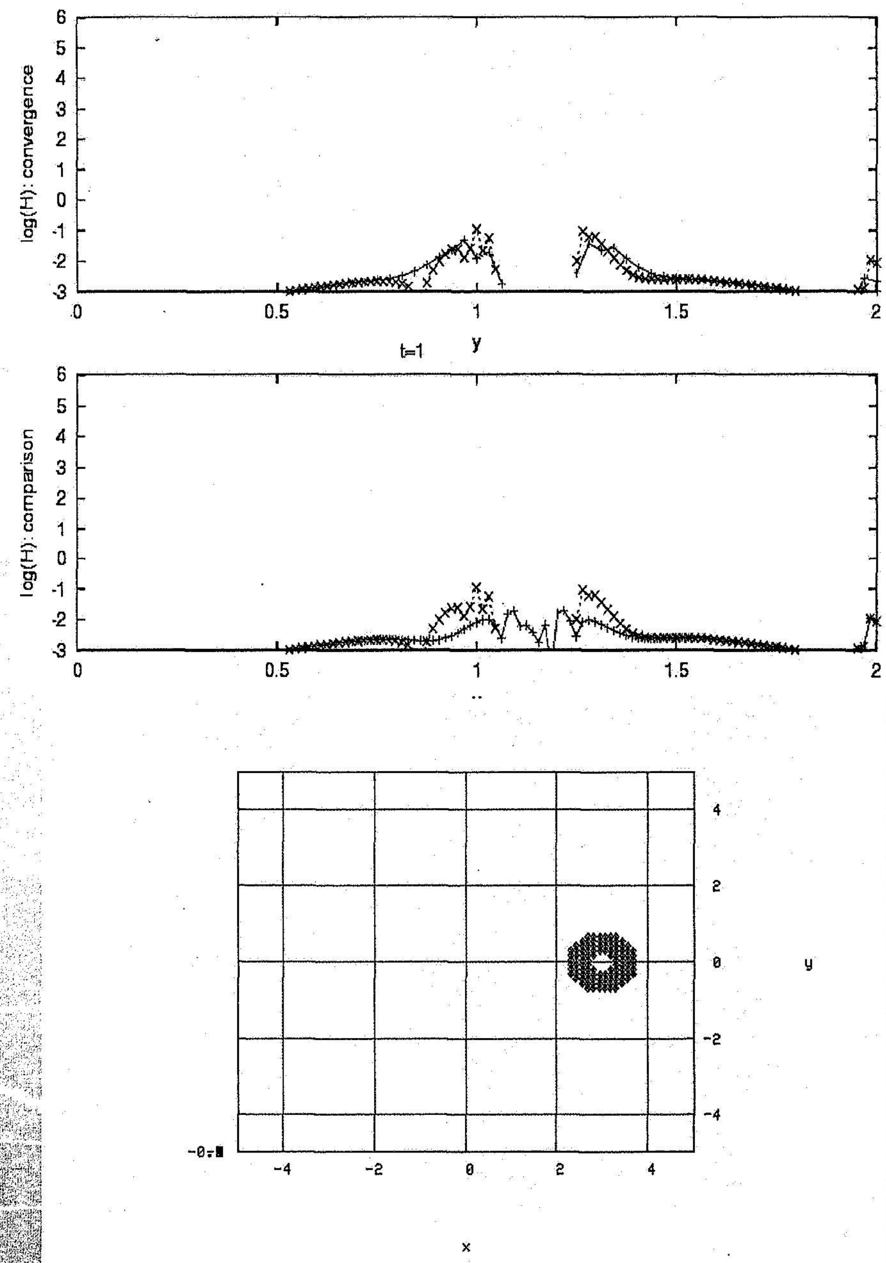
  - Simulations performed on Project Columbia supercomputer at Ames and other



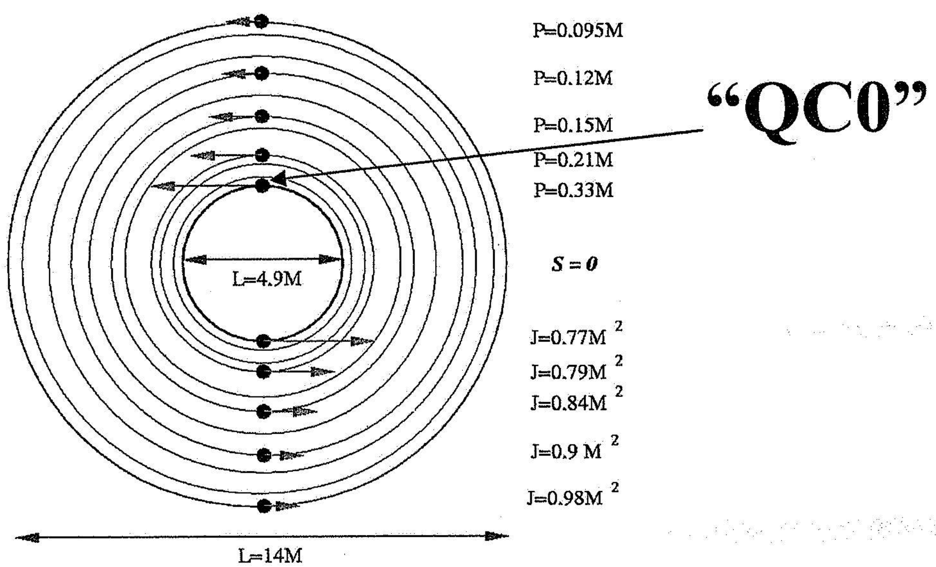


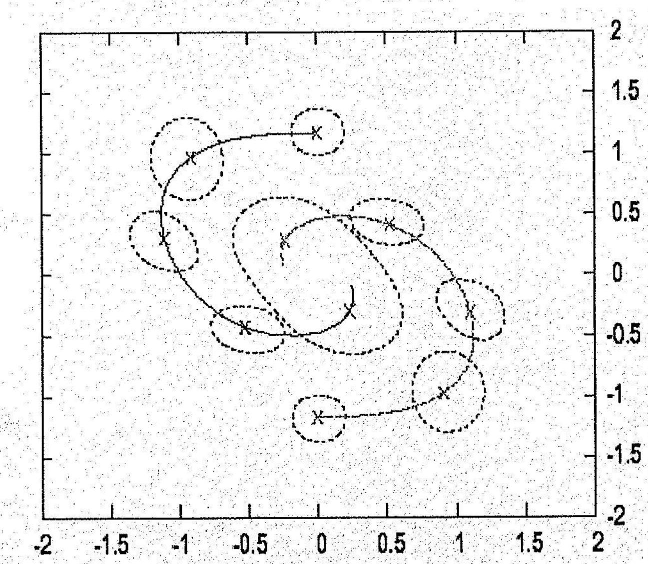
Goddard's Recent Advances

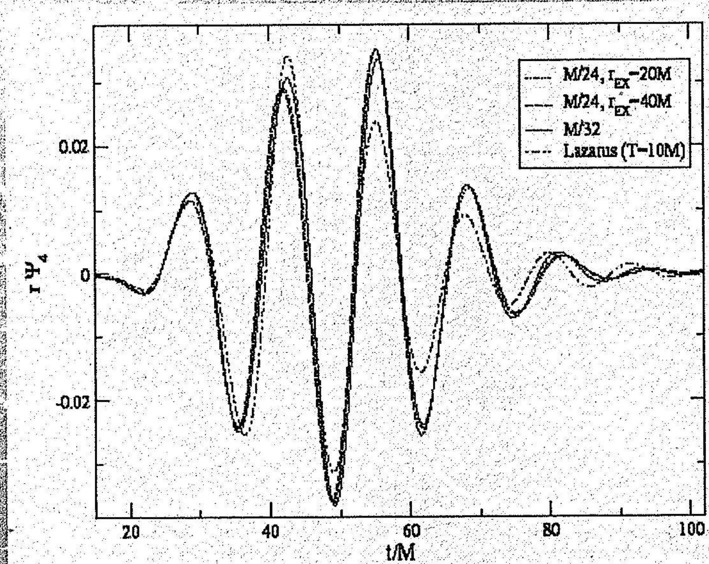
- **Old Problem**
  - The runs "crash" before merger
  - Large error develops near black holes
  - May be caused by fixing black holes to the grid
- **Solution**
  - Let the black holes move
  - New gauge (shift) condition
  - Modifed "puncture" black hole handling
  - Recent Summer/Fall focus of Goddard group



Baker, et al.: GSFC



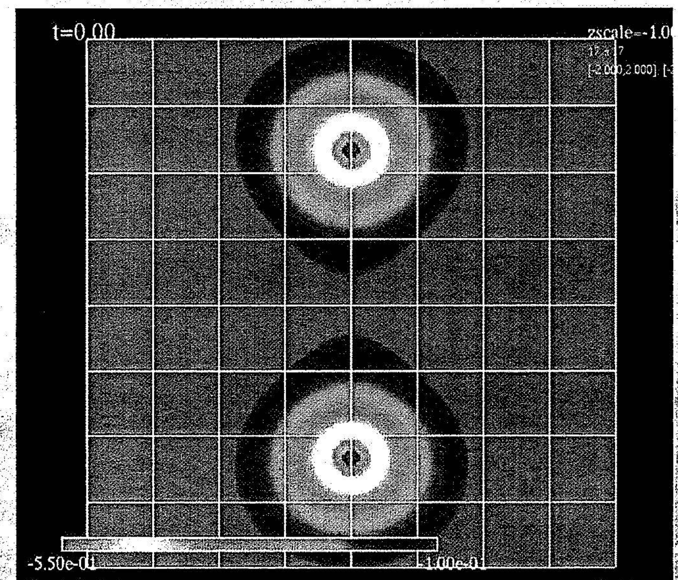


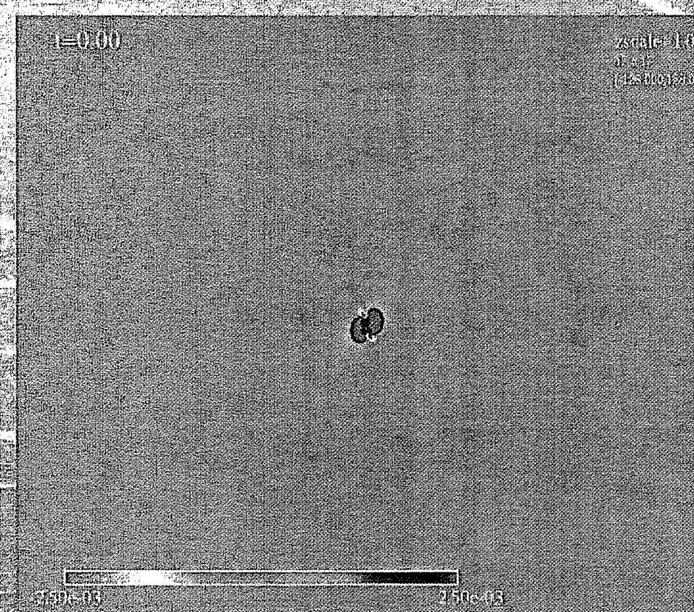


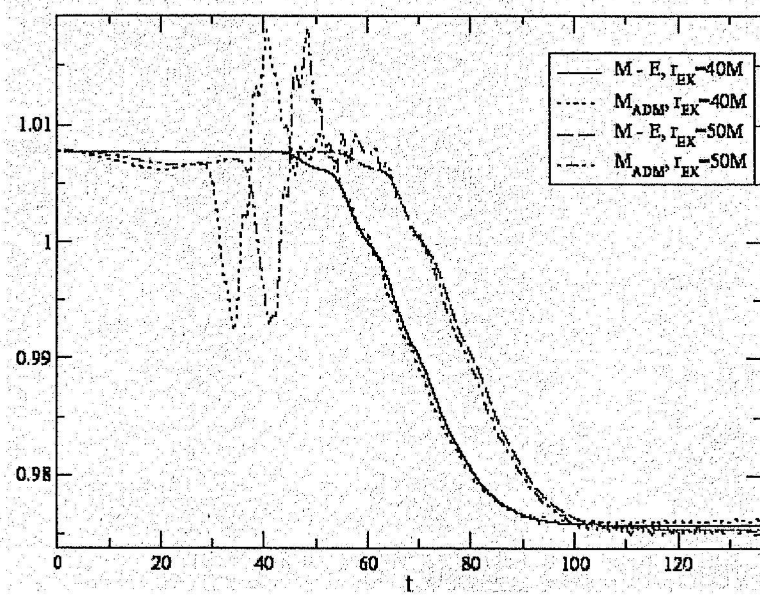
- Initial proper separation 5.0 M
- Top right: trajectories and apparent horizons at t = 0, 5*M*, 10*M*, 15*M*, 20*M*
- Bottom right:
  - l=2,m=2 waveforms at two resolutions, and extraction distance
  - also, compare with older Lazarus waveform

Baker, et al.: GSFC

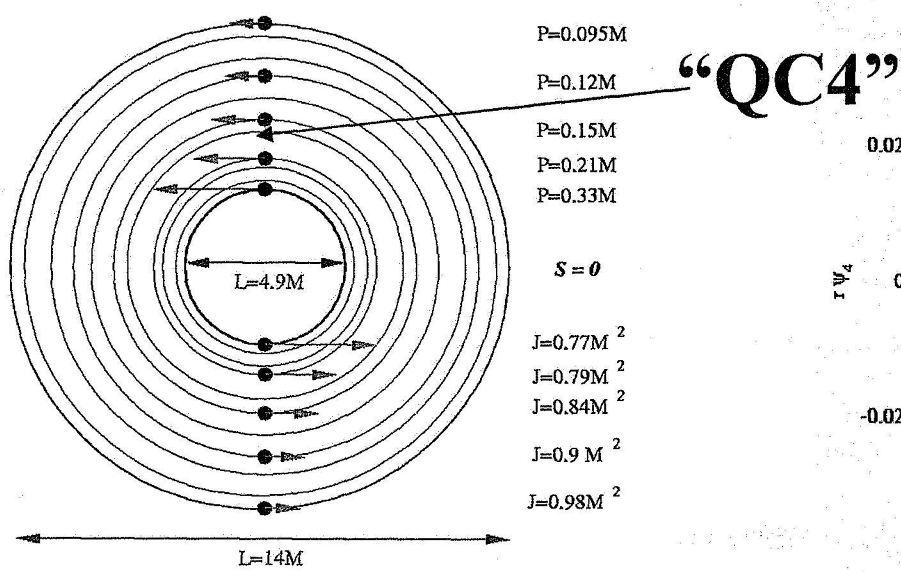
- Top right: evoln in curvature scalar
- Bottom right: gravitational waves
- Below: conservation test
  - $E = M - M_{\text{ADM}}$
  - extract at $40M$ (black) & $50M$ (red)
- Emits $E \sim 0.033M$
- Spin of final BH $\sim 0.65M$

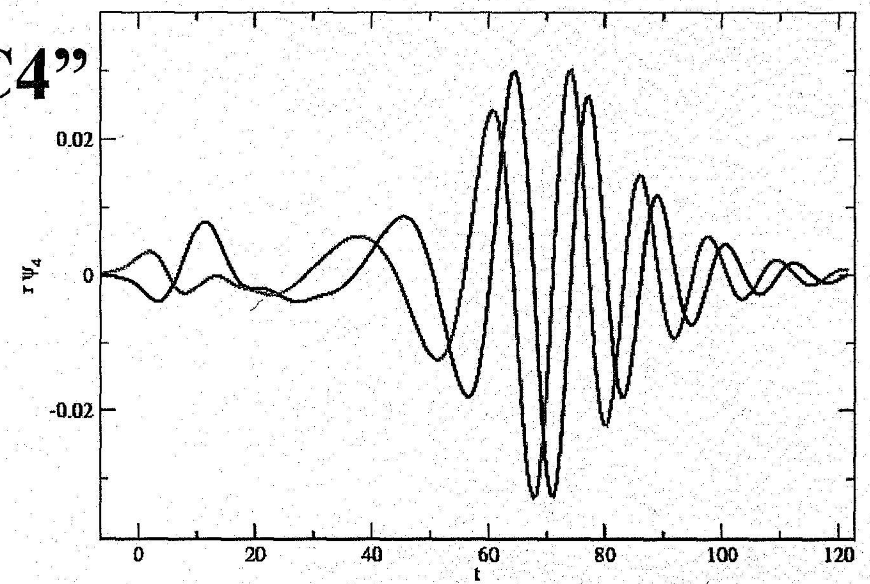




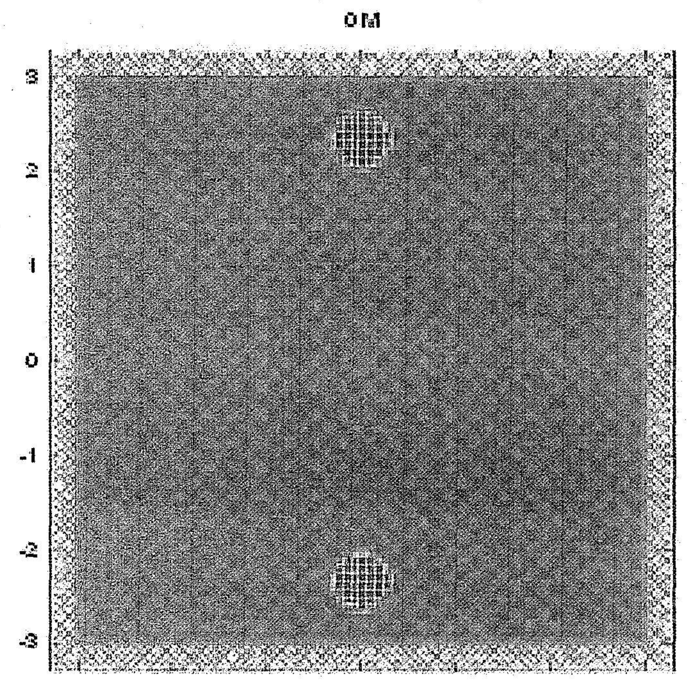


Starting farther out.



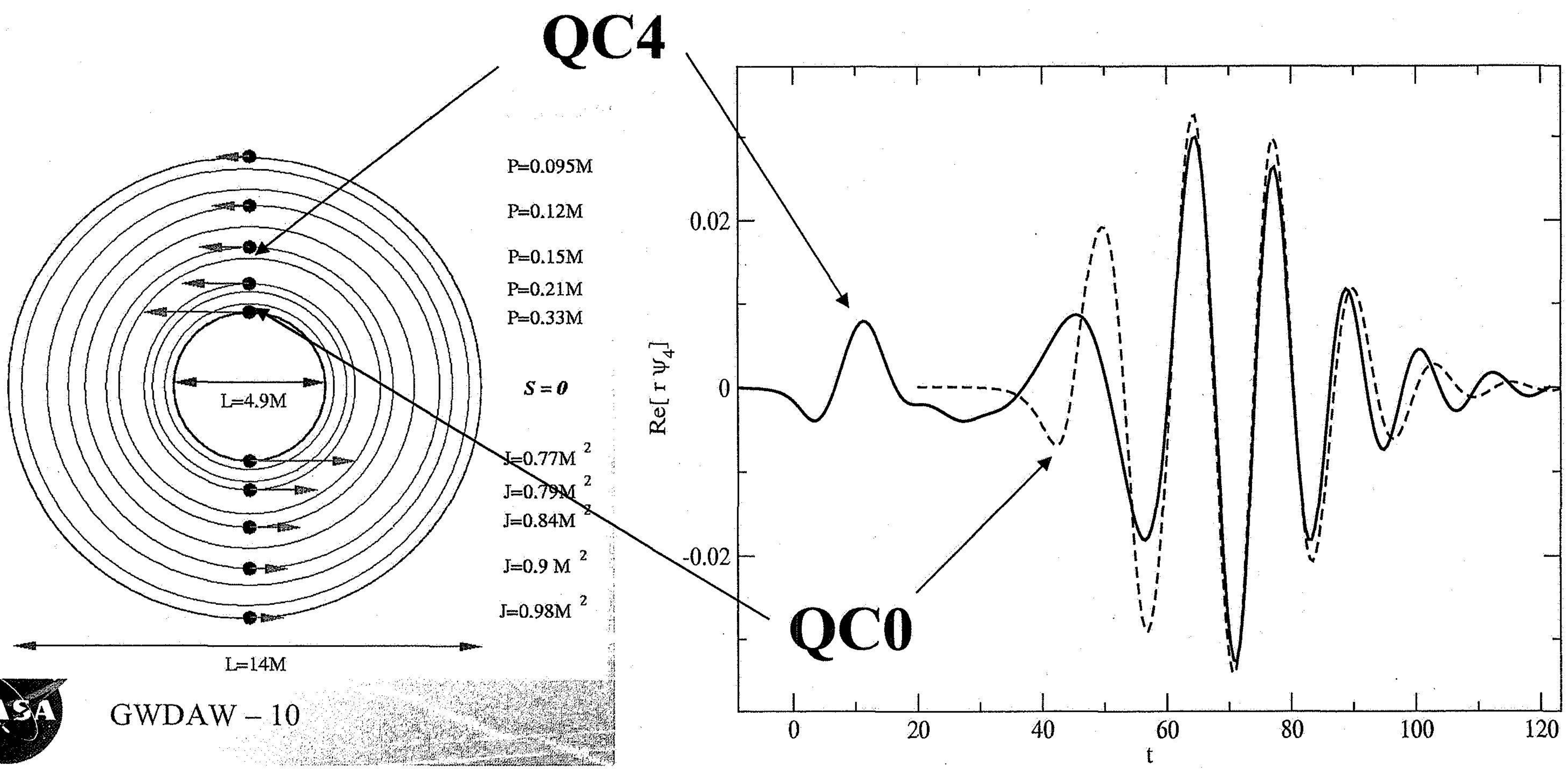


- New results (last week):
- Initial proper sepn 7.8M
- Evolves nearly 1 orbit before merger at ~ 55M
- Right: waveforms
- Below: evoln of apparent horizons



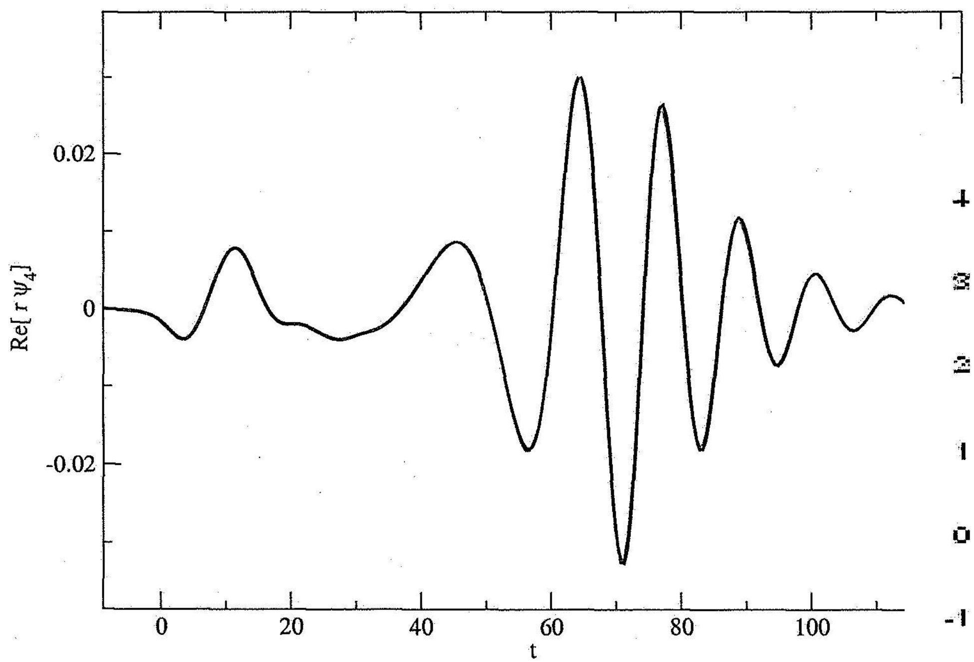
Comparing initial separation

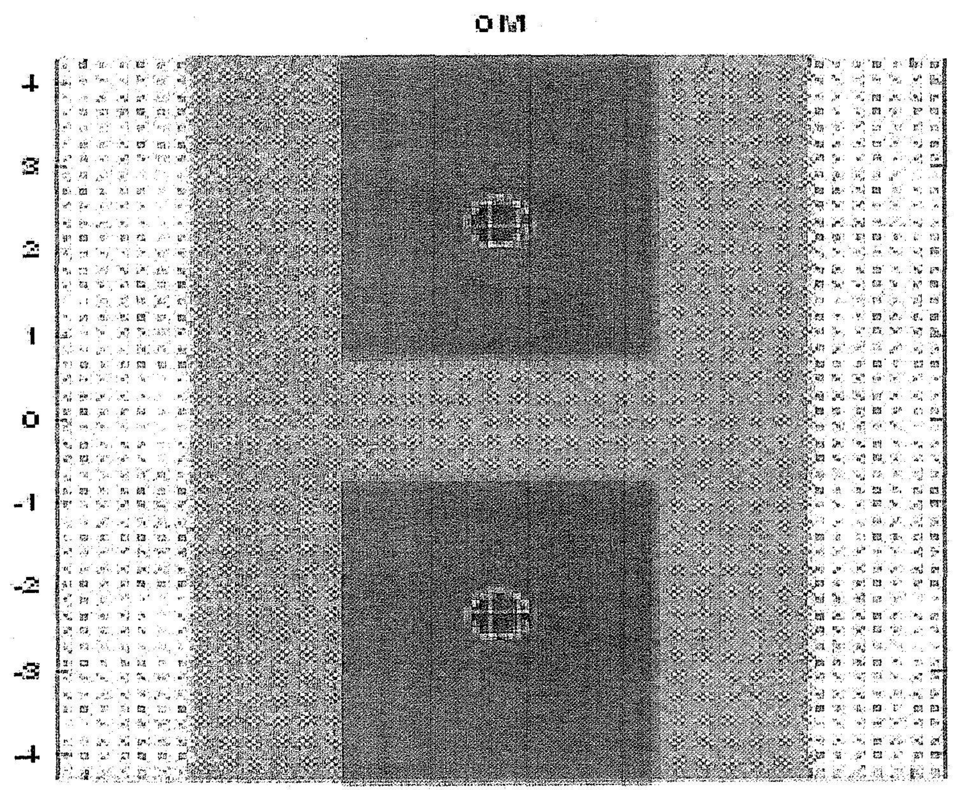
- Preliminary results
- Initial proper separation increased by 60%
- Uniform time and phase shift to compare for late time agreement
- Early part: initial data



Now with AMR

- *Really* new results (this week)
- QC4 data again
- Grid structure now adaptively tracking black holes
- More efficient, especially for farther separations





Conclusion

- ***Expect results from numerical relativity!***
- **Progress across a broad front:**
  - **multiple groups**
  - **independent codes**
  - **different approaches**
- **Groups are pushing forward quickly…**
  - **Begin exploration of parameter space: spin, nonequal masses…**
  - **Farther separations:**
    - **More astrophysical reliability**
    - **Connect to PN calculations**
- **LISA Data analysis**
  - **How much can this knowledge improve parameter estimation?**
  - **How can we formulate tests of GR?**
  - **What are critical questions for numerical investigation?**